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Distributed H_2/H_∞ Consensus Control for Multi-agent Systems with Directed Graph and Actuator Uncertainty

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Abstract

In this work, single-integrator multi-agent directed networks with energy bounded disturbances and Guassian white noises are considered in term of H_2 and H_∞ performance. Meanwhile actuator uncertainty is also the concern of H_∞ performance. Distributed control law of undetermined parameters is proposed. By using H_2/H_∞ theory and linear algebra, the problem is transformed into a non-convex optimization problem on diagonal matrices. Numerical iterative algorithms with consideration on both feasibility and conservatism are presented.

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Introduction

In last two decades, consensus control of multi-agent systems has been researched deeply and extensively, for its broad range of applications in several areas, such as: robotic teams cooperation, sensor networks, air vehicles formation flying, social networks and so on [20-24]. And it has attracted the attention from several fields including but not limited to engineering, biology, control theory and computer science.

For consensus problems, abundant achievements for first-order agents have been acquired by previous researchers [1-4]. The study of the multi-agent systems was from undirected graph to directed graph, from fixed communication topology to switched topology, and with time-delay or not. However, under real circumstances, the transmission errors, communication obstacles, actuator bias and other disturbances do exist. From now on, several researchers had investigated the disturbance rejection performance of multi-agent systems in different perspectives. P. Lin [5,6] firstly introduced H_∞ control theory to multi-agent systems. In his work, An orthogonal transformation was utilized to guarantee the validity of H_∞ control theory. And Y. Liu [7] investigated multi-agent systems of high-order integrator and general linear time-invariant dynamics by H_∞ criterion. Consensus protocols involving a coefficient were raised by Z. Li and Y. Zhao [9,10], and the H_∞ performance for both state feedback controlled systems and output feedback controlled systems was investigated by introducing H_∞ performance region of the coefficient. It is worthy mentioning that the authors in [9,29] addressed H_∞ and H_2 consensus problems of linear multi-agent systems and proposed algorithms to satisfy the H_∞ and H_2 conditions separately. Y. Cui [8] employed $L_2 - L_\infty$ theory to solve the peak bounded consensus problems of high-order multi-agent systems with external disturbances and parameter perturbations. In other perspectives, stochastic theory and probability limit theory were introduced to consensus problems of multi-agent systems [11,12], in both continuous-time and discrete-time case. V. Gupta [13] and Y. Jia [15] considered the LQG problems of networks of dynamical agents. When the systems are disturbed by Gaussian noises, the designs were synthesized under H_2 criteria for robust [25,26,31]. On another aspect of H_2 control, the recent paper [30] studied guaranteed cost problem for multi-agent systems with actuator faults and uncertainty.

In last century, numerous results were obtained in H_2/H_∞ control. DS. Bernstein [14] proposed a LQG performance approach with an H_∞ performance bound based on the solutions of three coupled Riccati equations. PP Khargonekar [16] simplified it to a convex sub-optimization problem. D. Arzelier [15] gave a sufficient condition in the form of bilinear matrix inequalities (BMIs) to reduce the conservatism, and solved this non-convex problem by a numerical iterative algorithm. C. Scherer [27] considered the more universal case where the channels of the disturbance inputs and controlled outputs are different, when different performance indexes are concerned. However, H_2 performance and H_∞ performance have never been considered together in multi-agent systems.

In this paper, consensus problems of single-integrator multi-agent systems with fixed directed interaction topology will be studied. We address the consensus performance subject to external L_2 disturbances and Guassian white noises by combining H_∞ theory and H_2 theory. Our objective is to search the linear feedback controller, based on relative states of its neighbors, to optimize the H_2 performance of the multi-agent systems, under two constraints. One is that the multi-agent systems reach asymptotically consensus with the absence of external signals, and the other is the constraint of H_∞ performance against both energy bounded disturbances and actuator uncertainties. Firstly, we will present the algorithms to design the satisfactory controller, rather than proposing a theorem to verify the protocol is valid or not. Secondly, different from other work, the feedback gain matrices of the multi-agent systems are Laplacian matrices with a fixed graphical structure constraint, which has more complicated relationship between the entries. We transform it to an optimization problem in a set of diagonal matrix. To measure the consensus H_2 performance, a novel output function is defined, which is linked to the LQR problem of linear-consensus. After a procedure of model transformation

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to guarantee the validity of H_∞ theory, an original elimination lemma is derived. By using this lemma, the problem is simplified to a numerically solvable BMI constrained optimization problem on diagonal matrices. Finally, a previously proposed iterative method is used to solve the BMI optimizaiton problem.

The paper is organized as follows. In section 2, some basic frameworks of graph theory and H_2/H_∞ control are introduced. H_2/H_∞ consensus problem for first-order directed network is described in section 3. Section 4 gives the main results and the approaches to solving the H_2/H_∞ consensus problem. In section 5, numerical simulations are presented to verify our results. Finally, conclusions are drawn in section 6.

Notations

In this paper, I_n represents the identity matrix with n dimensions, and 1_n represents n-dimension column vector $[1,1,...,1]^T$. Denote sym(A) = $A + A^T$, for a square matrix A. And $diag\{m_1,m_2,...,m_n\}$ is the diagonal matrix whose diagonal entries are given by $m_1,m_2,...,m_n$. For a n-order digraph, L_n denotes the set of all n-dimensional Laplacian matrices of the graph. And for a digraph with m edges, D_m denotes the set of all m-dimensional positive definite diagonal matrices. $L_2[0,\infty)$ represents the space of square integrable vector functions over $[0,\infty)$.

Preliminaries

Graph theory

Let $G = (v,\varepsilon,A)$ be a weighted directed graph of n orders, which consists of a set of nodes $V=\{v_1,v_2,...,v_n\}$, a set of edges $\varepsilon \subseteq V\times V$, And a weighted adjacency matrix $A=[a_{ij}]$. Node pair $(v_i,v_j) \in \varepsilon$ implies that there is a formation flow from v_j to v_i, where v_i and v_j call the tail of the edge and the head of the edge, respectively. The neighbor of vi is defined by $N_i=\{v_j \in V: (v_i, v_j) \in \varepsilon\}$. And the elements of adjacency matrix A are nonnegative, in which $a_{ij}>0$ if $v_j \in N_i$, and $a_{ij}=0$ otherwise. A sequence edges is called a directed path, if the tail of the latter edge is the same as the head of previous pair, such as $(v_{i_1},v_{i_2}),(v_{i_2},v_{i_3}),(v_{i_3},v_{i_4}),....$

The Lapliacan matrix Σ of a graph is defined as $L=[l_{ij}]_{m\times n}$, where $l_{ii}=\sum_{j=1}^{n} a_{ij}$ and $l_{ij}=-a_{ij}$ if $a\neq j$. Obviously, a Laplacian matrix will be a nonnegative definite matrix.

H_2 and H_∞ Theory

Consider the strictly proper system T_{zw} which yields to the following dynamics:

$$\dot{x} = Ax + Bw \\ Z = Cx \tag{1}$$

where A is a stable matrix.

Lemma 1

(Y. Jia [19]) The L_2 norm of strict proper system (1) satisfies $\| T_{zw} \|_2^2 = trace(B^T L_0 B)$, where Lo is the observability Gramian that satisfies

$$A^T L_0 + L_0 A + C^T C = 0 \tag{2}$$

Lemma 2

(Y. Jia [19]) Consider the system (1), the necessary and sufficient condition for $\| T_{zw} \|_\infty < \Upsilon$ is that the following Riccati equation has a positive definite solution

$$A^T P + PA + \Upsilon^{-2} PBB^T P + C^T C < 0 \tag{3}$$

Linear algebra

Lemma 3

(P. Lin et al. [5]) For matrix

$$C = \begin{bmatrix} \frac{n-1}{n} & -\frac{1}{n} & -\frac{1}{n} & \cdots \\ -\frac{1}{n} & \frac{n-1}{n} & -\frac{1}{n} & \cdots \\ \vdots & \ddots & \ddots & \ddots \\ -\frac{1}{n} & \cdots & -\frac{1}{n} & \frac{n-1}{n} \end{bmatrix}_{n\times n} \tag{4}$$

and an arbitrary n-order Laplacian matrix L, there exists an orthogonal matrix $U=[U_1,\bar{U}_1]$, where $\bar{U}_1=[1/\sqrt{n},...,1/\sqrt{n}]^T$, such that $U^T C U = \begin{bmatrix} I_{n-1} & 0 \\ 0 & 0 \end{bmatrix}$ and $U^T L U = \begin{bmatrix} U_1^T L U_1 & 0 \\ \bar{U}_1^T L U_1 & 0 \end{bmatrix}$.

Lemma 4

(CE. De Souza et al. [28]) Assume that D and E are real matrices with compatible dimensions. For an arbitrary scalar ε, we can get

$$DE + E^T D^T \leq \epsilon^{-1} DD^T + \epsilon E^T E \tag{5}$$

Lemma 5

(Schur complement) Given a symmetric matrix $S \in R^{n\times n}$ decomposed as

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

where $S_{11} \in R^{n\times n}$, $S_{12} \in R^{(n-r)\times n}$ and $S_{22} \in R^{(n-r)\times(n-r)}$. Then $S<0$ if and only if $S_{11}<0$ and $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$, or equivalently $S_{22} < 0$ and $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$

Problem Formulation

In [26,32], multi-UAVs (unmanned aerial vehicles) and satellite formations were supposed to disturbed by zero-mean Gaussian white noise. However, mathematically saying, the statistical characteristics of stochastic process may vary according to the information we have. In other words, the zero mean condition can not be guaranteed all the time. So we consider the singleintegrator multi-agent systems with identical dynamics, which are driven by energy bounded disturbances and white Gaussian noises.

$$\dot{x}_i(t) = u_i(t) + w_{\infty i}(t) + b_i w_{0i}(t) \tag{6}$$

where $x_i(t)$ is the state of the ith agent, $w_{\infty i}(t) \in L_2[0,\infty)$ is the energy bounded disturbance, and w (t) is the standard white Gaussian

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We propose the following protocol based on the states of neighbors to achieve consensus. However, multi-agent systems are always in the face of calculation errors and variations of communication circumstances. Different from other consensus scheme, the communication perturbation is reflected on the multiplicative uncertainty of weight elements, so H_∞ theory can be utilized.

$$u_i(t) = -\sum_{j\in N_i}[a_{ij}(1+\psi_{ij}(t))][x_j(t)-x_i(t)] \tag{7}$$

where a_{ij} is the parameter to be determined and $\psi_{ij}(t)$ is unknown multiplicative uncertainty of a_{ij} with

$$\psi_{ij}(t) = \begin{cases} \le \overline{\psi}_{ij} & if\ i \ne j\ and\ a_{ij} \ne 0 \\ 0 & otherwise, \end{cases} \tag{8}$$

In our work, the communication topology of the network is fixed and known. For example, it is like the layout of submarine optical cables and sensor networks, or some social relationship, in which the interaction topologies will not be changed over a period of time. Our objective is to find an optimal protocol to make the closed-loop system possess a desired level of disturbance rejection. For the H_∞ performance, on the one hand, the controlled output $z_{\infty i}(t)$ is present to value the difference between its state and the average state of all agents.

$$z_{\infty i}(t) = x_i(t) - \frac{1}{n}\sum_{j=1}^{n} x_j(t) \tag{9}$$

On the other hand, we expect that there is a tradeoff between the better system performance and the lower energy consumption. To consider the H_2 performance of the multi-agent system, a novel controlled output is given.

$$z_{0i}(t) = \sum_{j=1}^{n} q_{ij}(x_i(t)-x_j(t)) + r_i u_i(t) \tag{10}$$

where $q_{ij} > 0$ corresponding to an undirected complete graph and $r_i > 0$ are H_2 performance coefficients. On the one hand, denoting $u_t(t) = R^{-1}1_n 1_n^T x(t) + v(t) + \| T_{z_o w_0} \|_2^2 = \int_0^\infty [x^T(t)Q^2 x(t) + v^T(t)R^2 v(t)]dt$' which is the interaction free cost function of LQR-based optimization problem of linear-consensus in [17, 18], where $Q=[q_{ij}]n\times n$ and $R= diag\{r_1,...,r_n\}$. On the another hand, when $r_i = 0$ and Q=C, z_{0_i} can be used to measure the static output variance of consensus under Guassian white noises. While $\| T_{z_\infty w_\infty} \|_\infty$ is adopted to guarantee the energy amplification from the expectation of the noises to the expectation of consensus output.

By utilizing protocol (7), the closed-loop dynamics of whole multi-agent system can be written as

$$\begin{aligned} \dot{x}(t) &= -(L+\Delta L)x(t) + w_\infty(t) + B_{w_0}(t) \\ z_\infty(t) &= C_x(t) \\ z_0(t) &= (Q-RL)x(t) \end{aligned} \tag{11}$$

where $B = diag\{b_1, \ldots, b_n\}$, and C is defined in equation (4).

Problem 1

Given an H_∞ level γ, determine a Laplacian matrix L of the fixed interconnection of the multi-agent system (6) with multiplicative weight uncertainty satisfying (8), such that

$$J = \inf_{L\in L_n} \sup_{|\psi_{ij}(t)\le \overline{\psi}_{ij}|} \| T_{z_0 w_0} \|_2 \tag{12}$$

under

1. the closed-loop multi-agent system (11) achieves asymptotically consensus for arbitrary $\psi_{ij}(t)$ satisfied (8) with the absence of $w_\infty(t)$ and $w_0(t)$ and the arbitrary initial states $x(0)$.
2. $\| T_{z_\infty w_\infty} \|_\infty < \Upsilon$ i.e $\int_0^\infty z_\infty^T z_\infty - \Upsilon^2 w_\infty^T w_\infty dt < 0$ for $w_0(t) = 0$, initial state $x(0) = 0$ and arbitrary $w_\infty \in L_2[0,\infty)$ and $\psi_{ij}(t)$ satisfied (8).

Lemma 6

(W. Ren and RW. Beard [4]) The Laplacian matrix L of a directed graph G has a single zero eigenvalue (with eigenvector 1_n) if and only if the graph G contains a spanning tree. Moreover, other eigenvalues of L are strictly positive in this case.

In order to satisfy the first condition of Problem 1, it is a necessary condition in our work that the directed network of the multi-agent system contains a spanning tree.

Lemma 7

(P. Lin et al. [5]) Consider a directed graph G. Let $E = [e_{ij}]$ and $F = [f_{ij}]$ be the 01-matrix. The rows and columns of E are indexed by the nodes and edges, while the columns and rows of F are indexed by the nodes and edges. $e_{ij} = 1$ if the vertex i is the tail of the edge j, $e_{ij} = 0$ otherwise. And $f_{ij} = 1$ if the vertex j is the head of the edge i, $f_{ij} = 0$ otherwise. Let weight matrix $W = diag\{w_1, w_2, \ldots, w_{|\varepsilon|}\}$, where w_p ($p = 1, \ldots, |\varepsilon|$) is the weight of the pth edge of G and $|\varepsilon|$ is the number of the edges. Then the relationship of the Laplacian matrix can be obtained $L = EWD$, where $D = E^T - F$.

Main Results

In this section, some main results of this paper are presented.

Model Transformation

Because of the singularity of the system matrix $-(L + \Delta L)$, which leads to H_2/H_∞ theory invalid, some model transformations need to be conducted.

$$\begin{aligned} \hat{x}(t) &= x(t) - \frac{1_n}{n}\sum_{i=1}^{n}\int_0^t [w_\infty(s) + B_{w_0}(s)], \\ \delta(t) &= U_1^T \hat{x}(t) \text{ and } \overline{\delta}(t) = \overline{U}_1^T \hat{x}(t) \end{aligned} \tag{13}$$

Remark 1

Although $w_\infty(t)$ is a deterministic function with bounded norm while $w_0(t)$ is a random signal, sharing a common integral operator $\int_0^\infty$ will not create any confusions.

Then we can obtain that

$$\begin{aligned} \begin{bmatrix} \dot{\delta}(t) \\ \dot{\overline{\delta}}(t) \end{bmatrix} &= -\begin{bmatrix} U_1^T(L+\Delta L)U_1 0 \\ U_2^T(L+\Delta L)U_1 0 \end{bmatrix}\begin{bmatrix} \delta(t) \\ \overline{\delta}(t) \end{bmatrix} + \begin{bmatrix} U_1^T w_\infty \\ 0 \end{bmatrix} + \begin{bmatrix} U_1^T w_0 \\ 0 \end{bmatrix} \\ z_\infty &= [U_1 \quad 0]\begin{bmatrix} \delta(t) \\ \overline{\delta} \end{bmatrix} \\ z_0 &= [QU_1 - RLU_1 0]\begin{bmatrix} \delta(t) \\ \overline{\delta}(t) \end{bmatrix} \end{aligned} \tag{14}$$

Consider the H_2 and H_∞ performance $\| T_{z_0 w_0} \|_2$ and $\| T_{z_\infty w_\infty} \|_\infty$ of the multi-agent system, by Lemma 7, we study the reduced dimension system

$$T_{zw}^f(s) = \begin{bmatrix} -U_1^T EWDU_1 - U_1^T E\Delta W(t)DU_1 & U_1^T & U_1^T B \\ U_1 & 0 & 0 \\ QU_1 - REWDU_1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} A_f + \Delta A_f(t) & B_{\infty f} & B_{0f} \\ C_{\infty f} & 0 & 0 \\ C_{0f} & 0 & 0 \end{bmatrix} \quad (15)$$

Lemma 8

The following conditions are equivalent:

1. The protocol (7) can solve the Problem 1.
2. For a multi-agent system with digraph which contains a spanning tree, there exist symmetric positive definite matrices $\tilde{X}_2, X_\infty$ and a $|\varepsilon|$-dimensional diagonal positive definite matrix W solving the following non-convex optimization problem:

Problem 2

$$J = \min_{X_\infty, \tilde{X}_2 \text{ and } L \in L_n} \sup_{\psi_i(t)} trace(B_{of}^T \tilde{X}_2 B_{0f})$$

$$A_f^T \tilde{X}_2 + \tilde{X}_2 A_f + C_f^T C_f = 0 \quad (16)$$

$$\begin{bmatrix} (A_f + \Delta A_f)^T X_\infty + X_\infty (A_f + \Delta A_f) + C_f^T C_f X_\infty B_{\infty f} \\ B_{\infty f}^T X_\infty - \Upsilon^2 I \end{bmatrix} < 0 \quad (17)$$

And the cost function J is the Laplacian matrix L.

Proof Equation (16) can be obtained readily by the definition of L_2 norm in Lemma 1. And the H_∞ constraint can be transformed into inequlity (17) referred to [19].

Conditions for H_2/H_∞ Consensus

Lemma 9

(Elimination lemma) $P = [\bar{P}, -I], H$, H are given matrices with compatible dimensions, and H is a symmetric matrix. Denote $N_P = [I, \bar{P}^T]^T$.

Then, there exists a matrix $X = [X_1^T, X_2]^T$ where X_2 is a positive definite diagonal matrix, such that

$$H + P^T X^T + XP < 0 \quad (18)$$

if and only if

$$N_P^T H N_P < 0 \quad (19)$$

Proof Necessity can be obtained noticeably. Next, we use a constructive approach to prove the sufficiency. A full column rank matrix $\bar{V} = [0, I]^T$ is introduced to expand NP into a group of basis of the entire space, i.e. $V = [N_P, \bar{V}]$ is an invertible square matrix. So inequality (18) holds if and only if there exists X such that

$$V^T = (H + P^T X^T + XP)V < 0 \quad (20)$$

holds. According to the block of V, we assume that

$$V^T HV = \begin{bmatrix} H_{11} & H_{12} \\ H_{12}^T & H_{22} \end{bmatrix} \quad (21)$$

and obtain that

$$V^T XPV = \begin{bmatrix} 0 - X_1 & -\bar{P}^T X_2 \\ 0 & -X_2 \end{bmatrix} \quad (22)$$

Substituting equations (21) and (22) into inequation (20) so that it can be rewritten as

$$\begin{bmatrix} H_{11} & H_{12} - X_1 - \bar{P}^T X_2 \\ H_{12}^T - X_1^T - X_2 \bar{P} & H_{22} - X_2 - X_2 \end{bmatrix} < 0 \quad (23)$$

By Schur complement lemma, inequation (23) holds if and only if $H_{11} < 0$ and

$$H_{22} - X_2 - X_2 - (H_{12}^T - X_1^T - X_2 \bar{P}) H_{11}^{-1} (H_{12} - X_1 - \bar{P}^T X_2) < 0 \quad (24)$$

$H_{11} < 0$ is obtained by condition (19) and it is obvious that for matrix $X_1 = -\bar{P}^T X_2$, there exists a positive diagonal matrix X2 such that inequality (24) holds. So the sufficiency is proofed.

To simplify the conditions of optimization Problem 2, the following notations are stated:

$$N(X_2) = \begin{bmatrix} U_1^T Q^2 U_1 & -X_2 U_1^T E - U_1^T QRE \\ -E^T U_1 X_2 - E^T RQU_1 & E^T R^2 E \end{bmatrix}$$

$$L(X_\infty) = \begin{bmatrix} \mu U_1^T E_2^T E_2 U_1 + I & X_\infty & 0 & -X_\infty U_1^T E_1 \\ X_\infty & -\Upsilon^2 I & 0 & 0 \\ 0 & 0 & -\mu\psi^{-2} & 0 \\ -E_1^T U_1 X_\infty & 0 & 0 & 0 \end{bmatrix} \quad (25)$$

Theorem 1

Assume the directed graph of multi-agent system (6) contains a spanning tree, distribute H_2/H_∞ synthesis problem can be transformed to the following non-convex constraint optimization problem:

Problem 3

If there exist symmetric positive definite matrices X_∞, X_2, matrices K_{21}, $K_{\infty 1}$, $K_{\infty 2}$, $K_{\infty 3}$, diagonal matrices Y, Z, F_2, F_∞ and a positive scalar μ, such that

$$\hat{J} = \min_{\substack{\mu, X_2, X_\infty, K_{21}, K_{\infty 1}, K_{\infty 2}, \\ K_{\infty 3} \text{ and } Y, Z, F_2, F_\infty \in D_{|\varepsilon|}}} trace(B^T U_1 X_2 U_1^T B)$$

under

$$Z - F_\infty F_2^{-1} Y = 0 \quad (26)$$

$$N(X_2) + sym\left(\begin{bmatrix} K_2^T \\ -I \end{bmatrix} [YDU_1 \ F_2] \right) < 0$$

$$L(X_\infty) + sym\left(\begin{bmatrix} K_{\infty 1}^T \\ K_{\infty 2}^T \\ K_{\infty 3}^T \\ -I \end{bmatrix} [ZDU_1 \ -ZF_\infty] \right) < 0 \quad (27)$$

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It provides a sub-optimal solution to the H_2/H_∞ consensus problem and the upper bound $\hat{J}$ of the L_2 norm of the multi-agent system, by solving Problem 3.

At the optimum, the sub-optimal weight matrix can be given by

$$W^* = -F_\infty^{*-1} Z^* = -F_2^{*-1} Y^* \quad (28)$$

Proof By Lemma 4, the following inequality can be obtained for an arbitrary scalar $\mu > 0$

$$\Delta A_f(t)^T X_\infty + X_\infty \Delta A_f(t) \le \mu U_1^T D^T D U_1 + \frac{1}{\mu} X_\infty U_1^T E W \bar{\psi}^2 W E^T U_1 X_\infty \quad (29)$$

By Lemma 5, the H_∞ condition $\| T_{z_\infty w_\infty} \|_\infty < \Upsilon$ holds, if there exists a positive definite matrix X_∞ such that

$$\begin{bmatrix} -A_f^T X_\infty - X_\infty A_f + I_{n-1} + \mu U_1^T D^T D U_1 & X_\infty & X_\infty U_1^T D W \\ X_\infty & -\Upsilon I & 0 \\ W E^T U_1 X_\infty & 0 & -\mu \psi^{-2} \end{bmatrix} < 0 \quad (30)$$

Substitute $L(X_\infty)$ and $N(X_2)$ into the constraint (16) and (17). Then the H_2/H_∞ synthesis problem can be rewritten as

Problem 4

$$\min_{\mu, X_2, X_\infty, \text{ and } W \in D_{|\epsilon|}} trace(B^T U_1 X_2 U_1^T B)$$

under

$$[I U_1^T D^T W] N(X_2) \begin{bmatrix} I \\ W D U_1 \end{bmatrix} < 0 \quad (31)$$

$$\begin{bmatrix} I & 0 & 0 & U_1^T D^T W \\ 0 & I & 0 & 0 \\ 0 & 0 & I & -W \end{bmatrix} L(X_\infty) \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ W D U_1 & 0 & -W \end{bmatrix} < 0 \quad (32)$$

It indicates that the feasible region of W of Problem 4 is contained in the feasible region of Problem 2 by inequality (29). And we can readily get that $X_2 \ge \tilde{X}_2$ where X_2 is governed by inequality (31) while $\tilde{X}_2$ yields to equation (16). So it gives a upper bound of the L_2 norm of the multi-agent system by solving Problem 4.

Using Lemma 9 to slack some slack variables, we can obtain that Problem 4 equals to

$$\min_{\substack{\mu, X_2, X_\infty, F_{21}, F_{\infty 1}, F_{\infty 2}, \\ F_{\infty 3}, \text{ and } W, F_{22}, F_{\infty 4} \in D_{|\epsilon|}}} trace(B^T U_1 X_2 U_1^T B)$$

under

$$N(X_2) + sym\left(\begin{bmatrix} F_{21}^T \\ F_{22} \end{bmatrix} [W D U_1 \; -I] \right) < 0$$

$$L(X_\infty) + sym\left(\begin{bmatrix} F_{\infty 1}^T \\ F_{\infty 2}^T \\ F_{\infty 3}^T \\ F_{\infty 4} \end{bmatrix} [W D U_1 \; 0 - W - I] \right) < 0 \quad (33)$$

where $F_{22}, F_{\infty 4} \in D_{|\epsilon|}$ are positive definite diagonal matrices and other slack variables are real matrix with compatible dimensions. State that

$$Y = -F_{22} W \quad Z = -F_{\infty 4} W \quad K_2 = -F_{22}^{-T} F_{21}$$
$$K_{\infty 1} = -F_{\infty 4}^{-T} F_{\infty 1} \quad K_{\infty 2} = -F_{\infty 4}^{-T} F_{\infty 2} \quad (34)$$
$$K_{\infty 3} = -F_{\infty 4}^{-T} F_{\infty 3} \text{ and } F_2 = F_{22} \quad F_\infty = F_{\infty 4}$$

Since matrices Y, Z, F_2 and F_∞ are diagonal matrices with equality relationship (26), Problem 3 is obtained and Theorem 1 is proofed.

Remark 2

However, because of the nonlinear equational constraint (26), it is hard to utilize a numerical procedure to solve the constrained optimization Problem 3. We relax it by assuming that

$$F_\infty = F_2 = F \text{ and } Y = Z \quad (35)$$

Then we solve the following BMI constrained optimization problem

Problem 5

If there exist symmetric positive definite matrices X_∞, X_2, matrices K_2, $K_{\infty 1}$, $K_{\infty 2}$, $K_{\infty 3}$, diagonal matrices Z, F and a positive scalar μ solution of the optimization problem:

$$\min_{\substack{\mu, X_2, X_\infty, K_2, K_{\infty 1}, K_{\infty 2}, \\ K_{\infty 3}, \text{ and } Z, F \in D_{|\epsilon|}}} trace(B^T U_1 X_2 U_1^T B)$$

under

$$N(X_2) + sym\left(\begin{bmatrix} K_2^T \\ -I \end{bmatrix} [Z D U_1 F] \right) < 0$$

$$L(X_\infty) + sym\left(\begin{bmatrix} K_{\infty 1}^T \\ K_{\infty 2}^T \\ K_{\infty 3}^T \\ -I \end{bmatrix} [Z D U_1 \; 0 - Z \; F] \right) < 0 \quad (36)$$

to get an upper bound on the L_2 norm $\| T_{z_0 w_0} \|_2$. And optimal weight matrix $W^* = -F^{*-1} Z^*$.

A coordinate-descent based iterative procedure similar to [15] is used to solve the BML optimal problem.

Numerical Algorithm Description

Algorithm 1

1. Step 1: For a directed graph G which contains a spanning tree, choose the initial matrices $K_2 = K_{\infty 1} = W_0 D U_1$ and $K_{\infty 2} = K_{\infty 3} = 0$, where W_0 is an appropriate matrix with all eigenvalues positive.
2. Step 2k: Pick the optimal solution matrices K_2, $K_{\infty 1}$, $K_{\infty 2}$ corresponding to the minimum situation in the last step. Then solve the optimization linear matrix inequality (LMI) problem $\beta_{2k}^2 = \min_{\mu, X_2, X_\infty, \text{ and } Z, F \in D_{|\epsilon|}} trace(B^T U_1 X_2 U_1^T B)$ under inequality constraint (36)
3. Step 2k+1: Pick the diagonal optimal solution matrices Z, F corresponding to the minimum situation in the last step. Then solve the convex optimization LMI problem $\beta_{2k+1}^2 = \min_{\mu, X_2, X_\infty, K_2, K_{\infty 1}, K_{\infty 2}, K_{\infty 3}} trace(B^T U_1 X_2 U_1^T B)$ under inequality constraint (36)
4. Final step: If $\beta_{2k} - \beta_{2k+1} < \varepsilon$, then stop. The sub-optimal weight matrix of the digraph of the first-order multi-agent system is $W_\beta^* = -F^{-1} Z$ obtained, and the upper bound on the L_2 norm $\| T_{z_0 w_0} \|_2$ is obtained as $\beta_{2k+1}^{1/2}$; else go on to step 2k+2.

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Remark 3

From the description of the algorithm, it can be guaranteed that the target function will not increase at each iteration. It still will be a challenge task to proof the global convergency mathematically. However it has turned out effective in simulation and practice by choosing an appropriate initial value in the step 1 of Algorithm 1.

Numerical simulations

Consider a first-order multi-agent system with 5 agents. Their communication connection relationships are as Figure 1. Then we design a Laplacian matrix to solve the H_2/H_∞ sub-optimization problem. In other words, the weight of each edge need to be determined. By using Remark 2 and Algorithm 1, setting γ = 1, ε = 0.01 and $\bar{\varphi}_{ij} = 0.05$ then we can obtained the H_2/H_∞ sup-optimal solution W ∈ D_7 and L ∈ L_5, where W = diag{3.1043, 1.0915, 0.6263, 0.1846, 0.5366, 1.2051, 0.1403} and the parameters of protocol (7) and the upper bound $\beta_{2k+1}^{1/2} = 2.4685$ are also reaped.

$$L = \begin{bmatrix} 0.6769 & 0 & 0 & -0.5366 & -0.1403 \\ -3.1043 & 3.1043 & 0 & 0 & 0 \\ 0 & -1.0915 & 1.0915 & 0 & 0 \\ 0 & -0.6263 & -0.1846 & 0.8108 & 0 \\ -1.2051 & 0 & 0 & 0 & 1.2051 \end{bmatrix}$$

It is revealed from Figure 2 that the 5 agents can achieve consensus with good performance against Guassian white noises. And H_∞ performance is illustrated where $w_{\infty i}$ are sinusoidal disturbances in Figure 3. Figure 4 reveals the trajectory of the sum of the impulse response energy of z_0, which is equivalent to the L_2 norm of the closed-loop system $\| T_{z_0 w_0} \|$, and it is found approaching the upper bound we obtained $\beta_{2k+1}^{1/2}$ hereinbefore.

Compared with [5], our work goes a step further, not only because of the design of the Laplacian matrix we proposed, but also the consideration of H_2 performance. Using H_∞ approach in [5], assuming that the weight of each edge in graph 1 is equivalent, we can verify that the below equation; minimizes the L_2 norm of system (6) with $\| T_{z_\infty w_\infty} \|_\infty < 1$. And in this case, the sum of the impulse response energy of z_0 is shown in Figure 5.

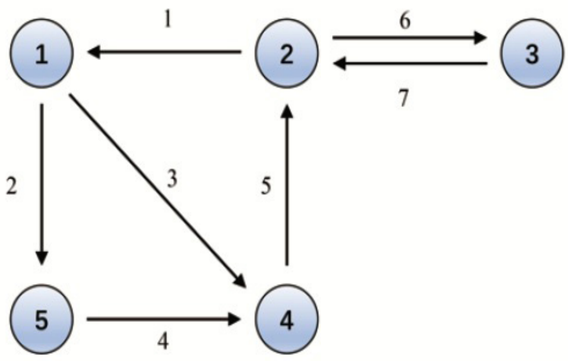

Figure 1: Topology 1 of the directed network of agents.

$$L = \begin{bmatrix} 2.762 & 0 & 0 & -1.381 & -1.381 \\ -1.381 & 1.381 & 0 & 0 & 0 \\ 0 & -1.381 & -1.381 & 0 & 0 \\ 0 & -1.381 & -1.381 & 2.762 & 0 \\ -1.381 & 0 & 0 & 0 & 1.381 \end{bmatrix}$$

Two extra examples are provided for comparison. The topologies of multiagent systems are shown in Figure 6 and Figure 7. H_2 performance results achieved by H_2/H_∞ approach and H_∞ approach are illustrated by Table. 1, which indicates that H_2/H_∞ approach can achieve a better robust H_2 performance compared with H_∞ approach with $\| T_{z_\infty w_\infty} \|_\infty < 1$.

Conclusion

This paper has studied the disturbance rejection problem of first-order multiagent system consensus with directed interaction topology. L_2 norm and H_∞ norm were utilized to measure the system consensus performance. Under the premise of satisfying given H_∞ performance, a new type of measure output contained more generality was proposed, and a relaxation approach with lower conservatism was presented to search the sub-optimal solution of the H_2/H_∞. Notably, a novel elimination lemma was obtained. In future work, high-order multi-agent systems with parameter uncertain and time-delay will be

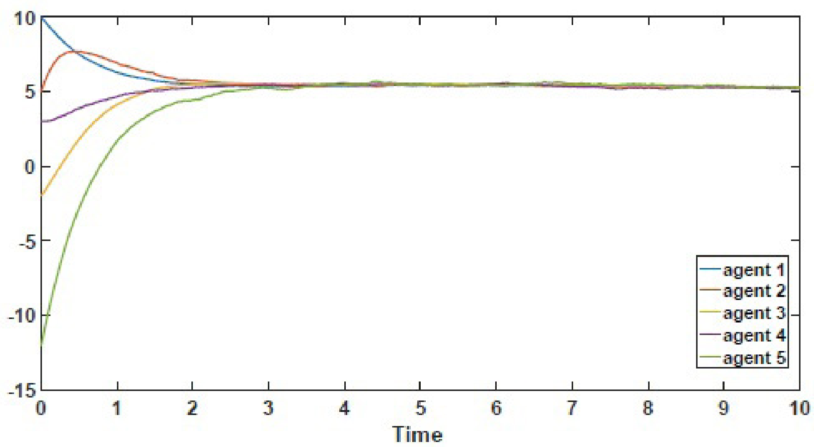

Figure 2: The state trajectories of the agents with Gaussian noises.

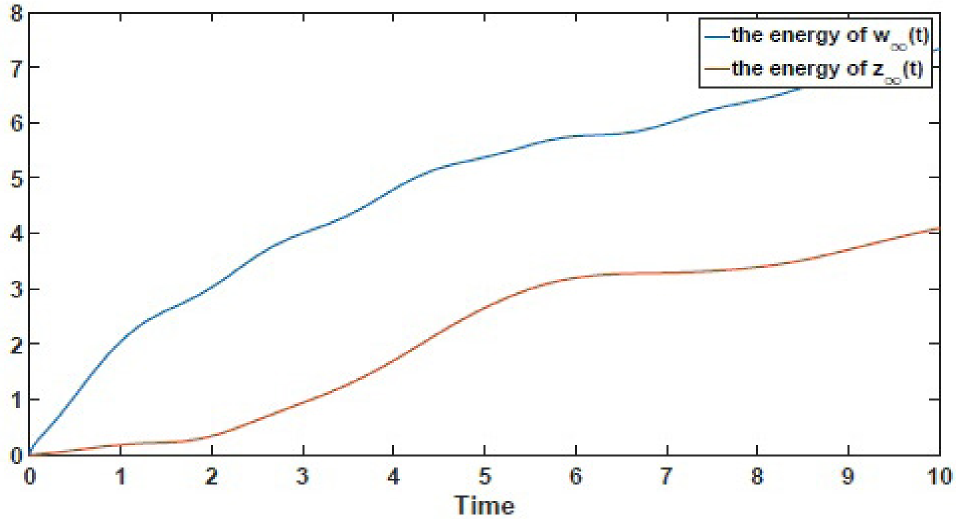

Figure 3: Energy trajectories of the energy bounded disturbance w∞ and the output z_∞

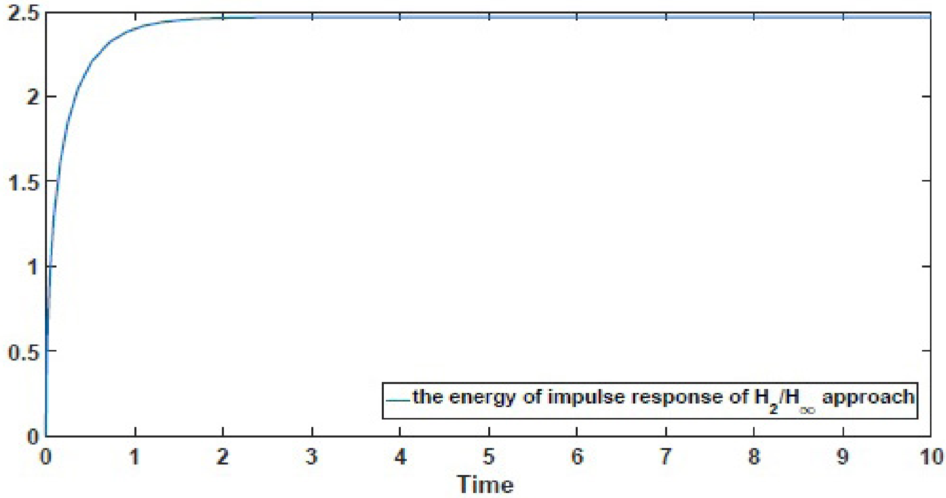

Figure 4: The sum of the impulse response energy of z_0 by H_2/H_∞ approach

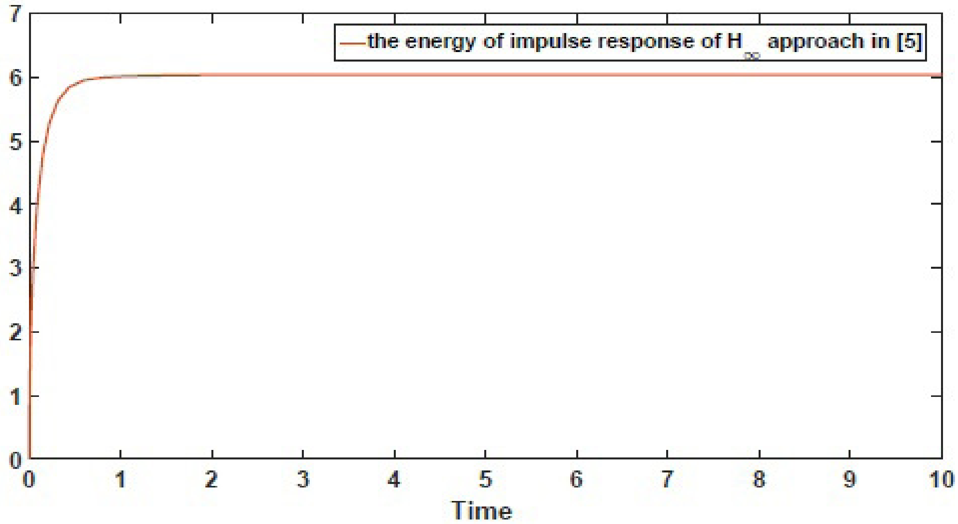

Figure 5: The sum of the impulse response energy of z_0 by H_∞ approach in [5].

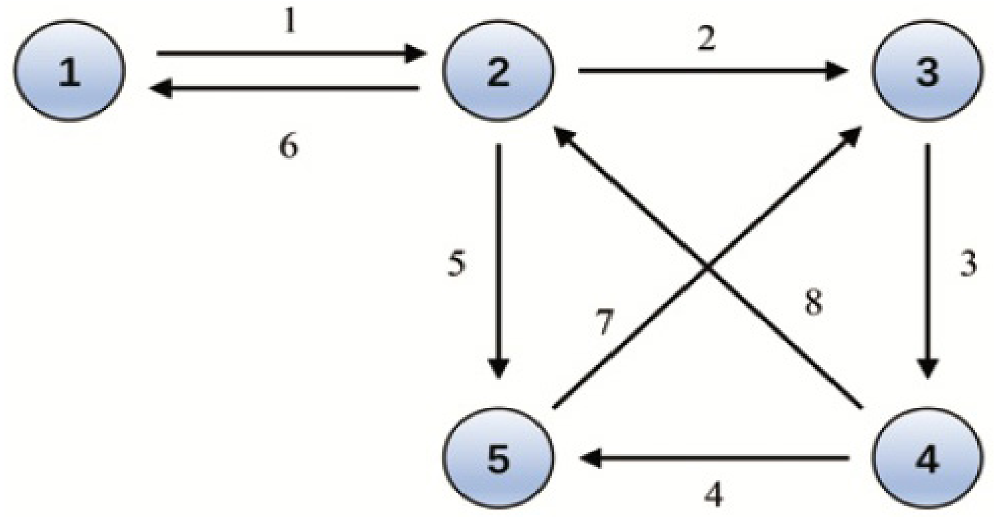

Figure 6: Topology 2 of the directed network of agents.

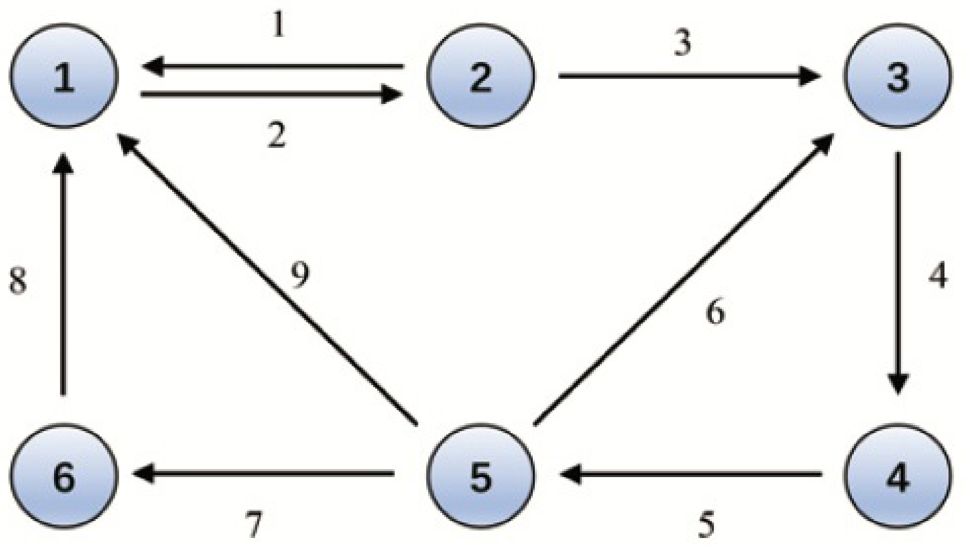

Figure 7: Topology 3 of the directed network of agents.

Multi agent system of	Sub-optimal L_2 norm by H_2/H_∞ approach	Sub-optimal L_2 norm by H_∞ approach in [5]
Topology 1	2.4685	6.0638
Topology 2	2.7202	4.3505
Topology 3	3.7241	4.7494

Table 1:

considered. For these cases, measurement errors caused by noises and higher conservatism caused by the larger number of the agents will also be the challenges to overcome.

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Competing Interests

The authors declare that no competing interests exist.

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