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Paraconsistent Sequential Linear-time Temporal Logic and Its Application to Clinical Reasoning Verification: A Brief Survey and Future Work

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Abstract

In this note, we present a brief survey of both paraconsistent sequential linear-time temporal logic and its application to clinical reasoning verication. Some recent works and plans for future work are also addressed. This note is mainly based on the papers [7, 8].

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Paraconsistent Sequential Linear-time Temporal Logic

Inconsistency-tolerant temporal reasoning with sequential (i.e., ordered or hierarchical) information is gaining increasing importance in computer science applications such as medical informatics and agent communication. Thus, a logical system for representing such reasoning is required to obtain a concrete theoretical basis for such applications. However, to the best of our knowledge, there are no good logical systems that can simultaneously represent inconsistency, sequentiality, and temporality. Thus, the aim of our work is to introduce a logical system, both semantically and syntactically, for appropriately representing inconsistency-tolerant temporal reasoning with sequential information.

Hence, we introduced and studied a new logic called *paraconsistent sequential linear-time temporal logic* (PSLTL) in [7, 8], which is an extension of the standard *linear-time temporal logic* (LTL) [15]. Inconsistency-tolerant reasoning in PSLTL is expressed via a paraconsistent negation connective, and sequential information is represented by sequence modal operators. Temporal reasoning in PSLTL is, of course, expressed by the temporal operators used in LTL. We showed that Kripke-style semantics for PSLTL are useful for appropriately handling clinical reasoning in a new model-checking framework called *paraconsistent (or inconsistency-tolerant) model checking*, where *model checking* is well-known to be a technology for verifying software [3]. We also proved some fundamental theorems for PSLTL, such as the completeness and cut-elimination theorems, which are obtained via theorems for semantically and syntactically embedding PSLTL into its fragments.

The proposed PSLTL is regarded as an extension of both LTL and *Nelson's paraconsistent four-valued logic with strong negation* N4 [1, 14]. On one hand, LTL is known to be one of the most useful temporal logics for verifying concurrent systems. On the other hand, N4 is known to be one of the most important base logics for inconsistency-tolerant reasoning. The combination of LTL and N4 was previously studied in [10], and such a combined logic was called *paraconsistent* LTL (PLTL). The combination of LTL with sequence modal operators was also previously studied in [12], and such a combined logic was called *sequence-indexed* LTL (SLTL). PSLTL is then obtained from PLTL by adding sequence modal operators and is also regarded as a modied paraconsistent extension of SLTL. Thus, PSLTL is a modied extension of both PLTL [10] and SLTL [12]. Moreover, we remark that we have recently introduced a new temporal logic called *sequential* LTL (sLTL) in [9], which is an improvement of SLTL.

Application to Clinical Reasoning Verication

In the following, we focus on the explanation of an important property of paraconsistent negation and some examples of clinical reasoning verication based on this property. As mentioned, the paraconsistent negation connective ~ used in PSLTL can suitably express inconsistency-tolerant reasoning. One reason why ~ is considered is that it can be added in such a way that the extended logic satises the property of *paraconsistency*. A semantic consequence relation |= is called paraconsistent with respect to ~ if there are formulas α and β such that $\{\alpha, {\sim}\alpha\} \not\models \beta$ In the case of PSLTL, this implies that there exist a model M and position i of a sequence $\sigma = t_0, t_1, t_2 \ldots$ of time points in M with $(M,i) \not\models (\alpha \wedge {\sim}\alpha) \to \beta$.

It is known that logical systems with paraconsistency can handle inconsistency-tolerant and uncertainty reasoning more appropriately than systems that are non-paraconsistent [16]. In [6], we considered clinical reasoning as such reasoning, where the example in [6] can also be handled in PSLTL. For example, we do not want $(s(x) \wedge {\sim}s(x)) \to d(x)$ to be satised for any symptom s and disease d, where ~s(x) means person x does not have symptom s and d(x) means person x suffers from disease d, because there may be situations that support the truth of both s(a) and ~s(a) for some individual a but do not support the truth of d(a).

In [6], we also considered another example. If we cannot determine whether someone is healthy, then the vague concept healthy can be represented by asserting the inconsistent formula *healthy(john)* ∧~*healthy(john)*. This is well-formalized in PSLTL because the

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formula *healthy(john)∧healthy(john)→hasCancer(john),* where has *Cancer(john),* which means John has cancer, is not valid in PSLTL (i.e., PSLTL is inconsistency-tolerant). On the other hand, the formula *healthy(john)∧¬healthy(john)→hasCancer(john)*, where ¬ is the classical negation connective, is valid in classical logic (i.e., inconsistency has undesirable consequences). For more information on paraconsistency and its applications, see, e.g., [16, 11, 6] and the references there in.

Recent and Future Works

We have recently proposed some PSLTL-based paraconsistent model checking examples in [13], wherein paraconsistent model checking and its applications to the verication of clinical reasoning and students' learning processes have been studied from the point of view of experiments. The standard model checkers SPIN [4] and NuSMV [2] have effectively been used in [13]. These experimental results are based on the translation algorithms and embedding theorems that were proposed in [6, 7, 8].

In the remainder of this note, we propose future work on clinical reasoning verification based on PSLTL. While state-of-the-art biology, it is still difficult to develop a new method of treatment in clinical medicine in a totally deductive manner. In general, some unverified ideas about the disease should be introduced by medical doctors who attempt to develop a novel treatment procedure on the basis of clinical reasoning. Although this difficulty for deduction originates from limitations of current biological understanding, some formal methods including PSLTL-based paraconsistent model checking can be provided to verify the procedure of a newly proposed treatment before its clinical trial by human subjects. Though formal verication by PSLTL-based paraconsistent model checking might have limited competence, we believe that such a formal verication framework should provide information about the robustness, toughness, and/or safety prior to the clinical trial.

The idea for verifying newly proposed treatment methods in clinical medicine can enhance another eld of medicine. One idea is the verication of a system of complementary medicine and/or alternative medicine. It would be benecial if the difference between traditional medicine (including experimental medicine) and complementary (alternative) medicine could be proven by using some formal methods including PSLTL-based paraconsistent model checking. Another idea is to check historical clinical trials by the same formal methods. Our questions for future directions in clinical reasoning verication based on PSLTL are as follows. What is the formal soundness of the famous rst small pox vaccine trial by Edward Jenner [5]? Have clinical trials grown more and more formally sound with the passage of time? We will try to answer these questions by using some PSLTL-based formal methods.

Competing Interests

The authors declare that there is no competing interest regarding the publication of this article.

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